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# Return Explanatory Ability and Predictability of Non-Linear Market Models

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# Return Explanatory Ability and Predictability of Non-Linear Market Models

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## Abstract

Recent literature supports the pricing of higher-order systematic co-moments of returns. This paper provides some support for the quadratic-market model that is consistent with the three-moment CAPM in explaining time-series returns of the winner and the smallest size portfolios. This study further uses three innovative methodologies in analysing the ability of the linear CAPM, the quadratic- and the cubic-market models in predicting one-period-ahead returns on individual stocks, equally- and value-weighted portfolios of momentum, size and country sorts. The results are surprising but important that the higher-moment CAPM market models do not outperform the linear CAPM in the return predictability tests.

*JEL Classifications:* G11, G12, G15

*Key Words:* Asset Pricing, Non-Linearity, Return Predictability

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# Return Explanatory Ability and Predictability of Non-Linear Market Models

## 1. Introduction

Asset pricing theory has identified the importance of the higher moments of return distributions beyond the variance in maximizing expected utility (Jean, 1971; and Scott and Horvath, 1980) and these insights have supported the use of higher-moment reformulations of the CAPM (Rubinstein, 1973). Despite the extensive evidence on the pricing of higher-order systematic co-moments and recent research into higher-moment CAPM market models, surprisingly little has been documented concerning the return predictability performance of these models. The purpose of this paper is to examine the relative performance of the higher-moment CAPM market models and the two-moment linear CAPM (Sharpe, 1964) in explaining contemporaneous returns and predicting one-period-ahead returns on individual stocks and various portfolio sorts. This paper is the first to investigate these issues and it contributes to the existing literature in the following three areas. First, the paper uses three innovative methodologies by which to evaluate the relative return predictability of the square (3-moment CAPM) and cubic (4-moment CAPM) market models. The methodologies involve the estimation of regressions of realised returns on the model predicted returns in the cross-section, time-series and pooled data. Second, the empirical tests are performed with returns on individual stocks as well as momentum, size and country portfolios with both equal- and value-weighting schemes. Third, the paper uncovers an interesting finding that the time-variation in realised returns on both the past "winner" and "smallest size" portfolios are associated with the market return in a non-linear manner.

The tests of the higher-moment CAPM market models on momentum portfolio returns are crucial since the momentum phenomenon documented by Jegadeesh and Titman (1993) remains difficult to explain (see, e.g., Fama and French, 1996; and Korajczyk and Sadka, 2004; among others). Also, recent research has suggested that the size and book-to-market factors of Fama and French (1996), which explain the size effect first documented by Banz ( 1981), may be proxies for higher-order systematic co-moments of returns (Chung et al., 2006). In addition, Hung (2007) shows that both momentum and size effects are partly attributable to coskewness and cokurtosis risks. Fuertes, Miffre and Tan (2005) also point out that momentum returns are related to non-normality risk.

Kraus and Litzenberger (1976) advocate the three-moment CAPM that incorporates the return distribution's third-order systematic co-moment (coskewness). A number of studies have provided evidence that coskewness helps explain the cross-section of stock returns (Friend and Westerfield, 1980; Barone-Adesi, 1985; Lim, 1989, Harvey and Siddique, 2000; Smith, 2007; and Errunza and Sy, 2005). Fang and Lai (1997) and Dittmar (2002) extend the analysis and present evidence of the pricing of fourth-order systematic co-moment (cokurtosis). Christie-David and Chaudhry (2001) also provide evidence for the pricing of cokurtosis in futures markets. To explain the time-series of returns, Kraus and Litzenberger (1976) derive a quadratic market model that is consistent with the three-moment CAPM, but without performing empirical tests. Hung, Shackleton and Xu (2004) show that the square and cube of the excess market return are modestly significant in explaining the size effect. In a similar vein, Ranaldo and Favre (2006) reported non-linear relationships between some hedge fund indices and market returns. Harvey, Liechty, Liechty and Muller (2004) evaluated the use of the higher moments of multivariate returns in portfolio selection and, on the basis of a Bayesian framework for incorporating the higher moments into the portfolio selection decision, demonstrated their importance in respect of maximizing expected utility. Davies, Kat, and Lu (2005) and Cremers, Kritzman and Page (2005) also demonstrated that the higher moments are particularly important in portfolio selection and allocation decisions of hedge funds since the return distributions associated with this asset class are typically highly skewed and leptokurtic (e.g., Brulhart and Klein, 2005).

The purpose of this paper is to provide further evidence on these issues by analysing non-linear market models using weekly return data that are typically even less normally distributed than monthly data (e.g., Chung, Johnson and Schill, 2006), but also provide a larger number of observations than is available from monthly data. Hence, our use of weekly data may provide significant advantages in terms of revealing the existence of any non-linear market return dependencies. The paper finds that, though the quadratic market model does not outperform the linear CAPM in terms of predicting one-period-ahead returns, it does contribute significant incremental explanatory power in respect of the ex post time-variation in returns on both the winner and the small size portfolios. In contrast, the cube of market return deviation does not explain a significant proportion of the return variations of any of the portfolio sorts. Overall, the evidence from the tests with returns on individual stocks, momentum, size and country portfolios confirms the above findings and the evidence is robust to both equally and value-weighted portfolios and portfolios constructed using either all stocks in the sample or the U.S. stock sub-sample. The discrepancy between the ex post explanatory power and ex ante predictive ability of the quadratic market model may be due to the parameter uncertainty which arises from the need to estimate unknown parameters from observed information (Williams, 1977; Breanan and Xia, 2001; and DeMiguel and Nogales, 2007) as well as the possibility of time-varying and unstable predictive relations (Paye and Timmermann, 2005; and Lewis, 2006).

The rest of this paper is structured as follows. Section 2 presents descriptive statistics of the sample. Section 3 specifies the cubic-market model and evaluates models in time-series of returns. Section 4 presents the methodologies for analysing the return predictability of the models. Section 5 reports the results of the return predictability tests. Section 6 provides robustness checks with the U.S. sample. This section further analyses a cubic model with orthogonal market terms and discusses parameter uncertainty. Section 7 concludes. The appendix summarises the quadratic and the cubic market models that are consistent with, respectively, the three- and four-moment CAPM.

## 2. Data and descriptive statistics of the sample

The empirical analyses focus on weekly U.S. dollar denominated stock returns (including dividends and capital gains). The market values (shares outstanding times prices) of the stocks are measured at the end of each week and the London Financial Times Euro dollar one-week rates (which serves as a proxy for the risk free rate) are collected from Datastream. The sample covers nineteen countries including Canada, the United States, Belgium, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Spain, Sweden, Switzerland, the United Kingdom, Australia, Hong Kong, Japan, Singapore and Taiwan and covers the 954-weeks from 22 September 1987 to 27 December 2005. The dataset includes both listed and delisted firms to mitigate any survivorship bias but excludes all non-common equities and companies listed outside of their domestic exchanges and all stocks with prices below $1[1]. To be included in the analysis, a stock had have both return and market value data for the respective analysis period. It is worth noting that the weekly returns are not contemporaneous across markets due to the different opening and closing times of each market.

[Insert Table 1 about here]

As shown in Panel A of Table 1, at the end of 2005 the sample covered 11,564 firms in total with the U.S. consisting of the largest number of stocks. The mean market value of companies in most of the countries increased from 1987 to 2005 while the median market

[1] The sample is very carefully screened by using all methods suggested by Ince and Porter (2006). The detailed procedures also involve checking company names to help verify their types and identifying geographical base, traded exchange name and traded currency for the common shares of each company. The padded zero return records at the end of each stock's history are also removed.

value of companies in some countries exhibited a decrease in 1996, which might be due to more company incorporations and listings of small size firms than those of large companies in early 1990's. Panel B of Table 1 reports descriptive statistics in terms of U.S. dollar returns of the value-weighted country portfolios constructed from the sample stocks in each country. The return distribution of the value-weighted global market portfolio constructed from all sample stocks has a mean weekly return of 0.17%, a standard deviation of 2%, a negative skewness of –1.33, a kurtosis of 14.71 and, according to the Jarque-Bera test, is significantly different from normal at 1% confidence levels. Indeed, the return distributions of all the nineteen countries are long-tailed (leptokurtic) and significantly different from normal at the 1% level. Panel C of Table 1 shows the descriptive statistics expressed in U.S. dollar returns of the equally-weighted country portfolios. The return distribution of the equally-weighted global market portfolio constructed from all sample stocks has a mean weekly return of 0.32%, a standard deviation of 1.68%, a negative skewness of –2.01, a kurtosis of 21.9, and is significantly different from normal at 1% confidence levels. As in the case of the value-weighted portfolios, the return distributions of all the nineteen equally-weighted country portfolios are significantly different from normal at the 1% level.

## 3. Time-series tests on return explanatory ability of models

### *3.1. Model specification*

This section examines whether the time-series variations in returns for the momentum and size portfolios are non-linearly associated with market returns using the following regression:[2]

$$R_{pt} - R_{ft} = C_{0p} + C_{1p}\left(R_{mt} - R_{ft}\right) + C_{2p}\left(R_{mt} - \overline{R}_{mt}\right)^2 + C_{3p}\left(R_{mt} - \overline{R}_{mt}\right)^3 + \varepsilon_t \qquad (1)$$

[2] Equation (1) gives definitions of security beta, gamma and delta that are consistent with the four-moment CAPM. Details are given in the appendix.

where $R_{pt}$ and $R_{ft}$ are respectively the return on portfolio $p$ and the risk-free asset at time $t$. $R_{mt}$ is the return at time $t$ on the value-weighted global market portfolio constructed from all sample stocks. The notation $\overline{R}_{mt}$ is the mean value of $R_{mt}$ for the entire sample period. All returns are expressed in U.S. dollar terms.

Table 2 shows that the mean excess market return, $(R_m - R_f)$, during the time-series test period is 0.08% per week. The excess market return is negatively correlated with the squared market return deviation with a statistically significant coefficient of –0.36. The excess market return is also significantly correlated with the cubed market return deviation $(R_m - \overline{R}_m)^3$ with a coefficient of 0.43. The squared and cubed market return deviations are negatively and significantly correlated with a correlation coefficient of –0.9. In Panel C of Table 2, the Spearman's rank correlation, which does not require the variables to be normally distributed, shows that the excess market return and the cubed market return deviation are highly correlated with a coefficient of 0.999. The correlation structure between the variables suggests the existence of multicollinearity. Thus, the magnitude of the coefficient estimates can change due to changes in the model specification such as the addition or deletion of an explanatory variable. In order to examine the robustness of the results with respect to collinearity, I further perform the tests using orthogonal market factors in Section 6.2.

[Insert Table 2 about here]

### *3.2. The dependent variables: returns on momentum and size deciles*

These tests focus on the representative momentum portfolios in the literature (e.g., Chordia and Shivakumar, 2002 and Rouwenhorst, 1998, among others) that rank and sort stocks into portfolios according to the past 6-month (or 24 weeks) compounded returns and then hold these portfolios for the 6 months following portfolio formation. Ten momentum portfolios are formed by ranking and sorting all the sample stocks based on their past 24-week compounded returns. The stocks within the top and bottom 10% of past returns comprise the 'winner' and the 'loser' portfolios, respectively. The formations of momentum portfolios are not overlapped in ranking periods in order to reduce trading frequencies and hence this implies substantially lower transaction costs associated with implementing this portfolio construction strategy[3]. Both equally- and value-weighted weekly portfolio returns are calculated in each of the 24 weeks following formations and then the portfolios are reconstructed. Size sorts occur every 48 weeks by ranking all stocks based on their market value of equity at the time of ranking. The small size portfolio ('small') and the big size portfolio ('big') contain stocks with the smallest and largest 10% of market capitalizations, respectively. Both equally- and value-weighted weekly portfolio returns are calculated in each of the 48 weeks following formations and then the portfolios are reconstructed. In total, each momentum and size portfolio has 930 observations of weekly portfolio returns from 8 March 1988 to 27 December 2005.

### *3.3. Time-series regression results*

Table 3 shows the regression results for the period from 8 March 1988 to 27 December 2005. The Newey-West standard errors are applied to correct for heteroskedasticity and autocorrelation of residuals. Panel A shows that $C_1$ is highly significant in every model for both the winner and the loser portfolios. The standard CAPM explains 55% of the return variations of the winner decile. The winner portfolio has lower market beta than the loser portfolio. For the winner portfolio, adding the squared market return deviation does increase model explanatory power as the slope coefficient $C_2$ is statistically significant and negative in Model 2 and Model 4 with slightly increased model adjusted $R^2$'s. The $F$-statistic, which tests whether the inclusion of the squared market return deviation to the linear CAPM increases the explanatory power of the model, is significant at the 1% confidence level.

[3] Lesmond, Schill and Zhou (2004) argue that overlapping strategies ignore the required high trading costs associated with high trading frequencies.

However, for the loser portfolio, the squared market term is insignificant and does not increase the explanatory power of models. The slope coefficient $C_3$ is statistically insignificant in both Model 3 and the cubic market model for both the winner and the loser portfolios. The cubed market term does not increase the explanatory power of models for both portfolios. Overall the results show that the squared market return deviation contributes incremental power toward explaining return variations of the winner portfolio. However, all the four models have significant intercepts for both the winner and the loser portfolios showing that these models cannot completely describe the return variations of the momentum portfolios.

[Insert Table 3 about here]

In explaining the time-series returns of the smallest and the biggest size deciles, Panel B shows that $C_1$ is highly significant in every model and is especially so for the biggest size decile. The linear CAPM has an insignificant intercept and explains 95% of the return variations for the biggest size decile. This is not surprising since, by construction, the value-weighted market portfolio and the biggest size decile portfolio are necessarily highly correlated, i.e., the value-weighted market portfolio is dominated by the returns of the largest firms. For the smallest size decile, the linear CAPM displays a significant intercept and a lower market beta with the $R^2$ of 30%. This result appears to confirm much previous research that has shown that the CAPM is generally a poor model in terms of explaining the returns on small size stocks (see, e.g., Banz, 1981).

The tests next examine whether the inclusion of the squared market return deviation increases the power of the models in terms of explaining returns. For the smallest size decile portfolio, the slope coefficient, $C_2$, of the squared market term is statistically significant and negative in both Models 2 and 4. The $F$-statistic is significant at the 1% level for the inclusion of the squared market return deviation. However for the biggest size decile, the slope coefficient, $C_2$ is statistically insignificant. The inclusion of the cubed market term does not appear to exert any significant influence in respect of explaining the returns of either portfolio as neither of the model estimates have significant $C_3$ coefficients. Overall, the linear CAPM explains the time-variation in returns on the biggest size decile relatively well. The squared market return deviation contributes incremental power toward explaining the time-variation in returns of the small size stock portfolio. However, all four models have significant intercepts which indicates that these models are unable to completely describe the return variations arising from size-sorted portfolios.

## 4. Return predictability tests of non-linear market models

This section performs three tests to study the relative performance of the higher-moment CAPM market models and the linear CAPM in predicting one-period-ahead returns for individual stocks, momentum, size and country portfolios. The market returns in all tests in this section are U.S. dollar returns of the value-weighted global market portfolio constructed from all sample stocks. For the tests with individual stocks, each stock must have at least 150 weekly returns in the coefficient estimation period and 12 weekly return observations following the estimation period. The tests are also performed on 100 momentum and size portfolios. For momentum portfolios, all stocks are ranked based on their past 24-week compounded returns and then sorted into 100 portfolios. Both equally- and value-weighted weekly portfolio returns are calculated for each of the 24 weeks following formations and then the portfolios are reconstructed. The formations of momentum portfolios are not overlapping in stock ranking periods. For the size portfolios, all stocks are ranked based on their market capitalisations at the time of ranking and then sorted into 100 portfolios. Both equally- and value-weighted weekly portfolio returns are calculated for each of the 48 weeks following formations and then the portfolios are reconstructed. Each momentum and size portfolio has a total of 930 weekly return observations from 8 March 1988 to 27

December 2005. In addition, the 19 country portfolios using both equally- and value-weighting schemes, are formed and tested.

The tests involve three stages. The first two stages that estimate model parameters and predict asset returns are the same for all the three tests. The only difference in tests occurs in the third stage where either cross-sectional, time-series or pooled regressions of *realised* returns on model *predicted* returns are undertaken. Firstly in each period $t = \tau$ for each risky asset, the intercept and slope coefficients of Equation (1) are estimated on a rolling basis from a time-series regression by using the previous 150 weeks of returns from $\tau - 149$ to $\tau$. In the second stage, the parameter estimates of the model are used to obtain the one-period-ahead return for each risky asset by incorporating realised returns on the global market portfolio and the risk-free rate at time $t$ + 1 according to:

$$\hat{r}_{p,t+1} = C_{0p} + C_{1p}\left(R_{m,t+1} - R_{f,t+1}\right) + C_{2p}\left(R_{m,t+1} - \overline{R}_{m,t+1}\right)^2 + C_{3p}\left(R_{m,t+1} - \overline{R}_{m,t+1}\right)^3 \qquad (2)$$

where $\hat{r}_{p,t+1}$ is the *predicted* return on asset $p$, $R_{m,t+1}$ and $R_{f,t+1}$ are *realised* return on the global market portfolio and the risk-free rate at time $t$ + 1, respectively. The notation $\overline{R}_{m,t+1}$ is the mean value of $R_m$ for up to time $t$ + 1.

The parameter estimation and return prediction procedures are also performed for the quadratic-market model as in (3) and (4) and for the linear-market model as in (5) and (6):

$$R_{pt} - R_{ft} = C_{0p} + C_{1p}\left(R_{mt} - R_{ft}\right) + C_{2p}\left(R_{mt} - \overline{R}_{mt}\right)^2 + \varepsilon_t \qquad (3)$$

$$\hat{r}_{p,t+1} = C_{0p} + C_{1p}\left(R_{m,t+1} - R_{f,t+1}\right) + C_{2p}\left(R_{m,t+1} - \overline{R}_{m,t+1}\right)^2 \qquad (4)$$

$$R_{pt} - R_{ft} = C_{0p} + C_{1p}\left(R_{mt} - R_{ft}\right) + \varepsilon_t \qquad (5)$$

$$\hat{r}_{p,t+1} = C_{0p} + C_{1p}\left(R_{m,t+1} - R_{f,t+1}\right). \qquad (6)$$

*4.1 Cross-sectional regressions of realised returns on predicted Returns*

Having obtained the predicted returns from the first two stages, this third stage test performs *cross-sectional* regressions of *realised* excess returns on *predicted* returns for risky assets in each period to examine the return predictability of models according to:

$$\tilde{r}_{p,t+1} = \lambda_0 + \lambda_1 \hat{r}_{p,t+1} + \tilde{\varepsilon}_p \tag{7}$$

where $\tilde{r}_{p,t+1}$ and $\hat{r}_{p,t+1}$ are respectively the *realised* excess return and *predicted* return on asset $p$ at time $t + 1$. $\tilde{\varepsilon}_p$ is the residual term across *assets*.

The above regression examines whether the *cross-sectional* variability in model predicted returns explains realised asset returns. Once the intercept and the slope coefficient of (7) for each *cross-sectional* period are obtained, they are then averaged across all *periods* as:

$$\lambda_j = \frac{1}{T}\sum_{t=1}^{T}\lambda_{jt} \tag{8}$$

where $\lambda_{jt}$ is the parameter estimate for period $t$ and $T$ is the number of *cross-sectional periods* in the sample; $j$ = 0 for the intercept and 1 for the slope coefficient.

The $p$-value for testing the significance of each parameter is the $p$-value corresponding to the $t$-statistic that is calculated by the mean of the parameter divided by its standard error,

$$t_{\lambda j} = \frac{\lambda_j}{std_{\lambda j} / \sqrt{T}} \tag{9}$$

where $std_{\lambda j}$ is the standard deviation of $\lambda_j$.

If the cross-sectional variation in model predicted returns explains one-period-ahead realised returns, the intercept $\lambda_0$ should be insignificantly different from zero, the coefficient $\lambda_1$ should be close to unity and significantly different from zero. However, the model adjusted $R^2$ will still be lower than 100% due to idiosyncratic risk of assets. Thus, by comparing the coefficient significance of $\lambda_0$ and $\lambda_1$ and the adjusted $R^2$s of the different models, it is possible to infer whether the higher-moment CAPM market models have a greater ability in predicting one-period-ahead asset returns than the linear CAPM.

## *4.2 Time-series regressions of realized returns on predicted returns*

This test performs the third stage *time-series* regression of realised excess returns on predicted returns for each risky asset over the entire period according to:

$$\tilde{r}_{p,t+1} = \lambda_0 + \lambda_1 \hat{r}_{p,t+1} + \tilde{\varepsilon}_{t+1} \tag{10}$$

where $\tilde{r}_{p,t+1}$ and $\hat{r}_{p,t+1}$ are respectively the *realised* excess return and *predicted* return on asset $p$ at time $t + 1$. $\tilde{\varepsilon}_{t+1}$ is the residual over time for *a single asset*.

The above *time-series* regression examines whether the predicted one-period-ahead returns explains realised returns for each individual asset. Once the intercept and the slope coefficient of (10) are obtained for each asset in the sample, they are then averaged across *assets* as:

$$\lambda_j = \frac{1}{N}\sum_{p=1}^{N}\lambda_{jp} \tag{11}$$

where $\lambda_{jp}$ is the parameter estimate for asset $p$ and $N$ is the number of assets in the sample; $j$ = 0 for the intercept and 1 for the slope coefficient.

The $p$-value for testing the significance of each parameter is the $p$-value corresponding to the $t$-statistic that is calculated by the mean of the parameter divided by its standard error,

$$t_{\lambda j} = \frac{\lambda_j}{std_{\lambda j}/\sqrt{N}} \tag{12}$$

where $std_{\lambda j}$ is the standard deviation of $\lambda_j$.

## *4.3 Pooled regressions of realised returns on predicted returns*

This final third stage test consists of a pooled regression of realised excess returns on predicted returns for all risky assets. This method has the advantage of avoiding averaging coefficients and the adjusted $R^2$s over cross-sectional periods or across assets and thus is expected to produce more precise results than the previous two methods.

The pooled regressions are estimated according to Equation (13) for the entire sample period:

$$\tilde{r}_{p,t+1} = \lambda_0 + \lambda_1 \hat{r}_{p,t+1} + \tilde{\varepsilon}_{p,t+1} \tag{13}$$

where $\tilde{r}_{p,t+1}$ and $\hat{r}_{p,t+1}$ are respectively the *realised* excess return and *predicted* return on asset $p$ at time $t$ + 1, $\tilde{\varepsilon}_{p,t+1}$ is the residual across assets and over time periods. The residuals are assumed to be independent of the predicted returns and have a zero mean with finite variance as discussed by Sayrs (1989) and Petersen (2007), but are not assumed to be independently and identically distributed. The standard errors are corrected for within-cluster correlation and heteroskedasticity using the methods of Petersen (2007).

# 5. Results of return predictability tests

## *5.1 Cross-sectional results of realised returns on predicted returns*

### *Individual stocks*

Table 4 shows the results of cross-sectional tests in respect of the individual stock empirical estimates. There are 12,262 firms that have at least 162 weeks return histories in the sample. For a stock that has a complete return history throughout the 930-week period from 8 March 1988 to 27 December 2005, the first set of coefficient estimates $C_{0p}$, $C_{1p}$, $C_{2p}$ and $C_{3p}$ of Equations (1) is estimated at 15 January 1991 and the final set is estimated at 20 December 2005. The maximum number of coefficient sets for each stock is 780. These estimates are carried forward to the next period to predict stock returns during the 780 weeks from 22

January 1991 to 27 December 2005. In the third stage, cross-sectional regressions as detailed by Equation (7) of realised excess returns on predicted returns are performed in each week.

[Insert Table 4 about here]

The results show that the linear market model has the smallest, but still significant, intercept $\lambda_0$, a highly significant $\lambda_1$ coefficient of 0.39 and the highest adjusted $R^2$ among all three models. The cubic market model produces the smallest but significant $\lambda_1$ and the lowest adjusted $R^2$ among the three models. Overall, the linear market model appears to provide the best performance in terms of predicting the one-period-ahead stock returns.

The low adjusted $R^2$s of all models may be due to three reasons. The first possible explanation is the high level of idiosyncratic risk associated with individual stocks. The second candidate is the errors-in-variables (EIV) problem pointed out by Kim (1997). The coefficient estimates ($C_0$, $C_1$, $C_2$ and $C_3$) in the first stage for individual stocks may be excessively noisy, which leads to imprecise estimates of predicted returns in the second stage. Hence, the EIV problem reduces the ability of the models to predict one-period-ahead return and also produces downward biases to the magnitude of $\lambda_1$. Finally, the predicted returns may be centring on the mean returns implied by the models and thus the variability of predicted returns is lower than that of the realised stock returns. Consequently, the ability of the models to predict one-period-ahead return is reduced. Examining model performance with portfolio returns may help to cure these potential problems because the formation of portfolios largely removes idiosyncratic risk and reduces the noisy components in the coefficient estimates. In addition, the variability of realised portfolio returns is smaller than that of individual stocks.

*Portfolios*

There are 930 observations of weekly portfolio returns from 8 March 1988 to 27 December 2005 for each of the 100 momentum and size and 19 country portfolios. The intercept and slope coefficients of Equations (1), (3) and (5) are estimated every week for each portfolio on a rolling basis during the 780 weeks from 15 January 1991 to 20 December 2005. Once the coefficients are estimated, they are used to predict the one-period-ahead portfolio returns during the 780 weeks from 22 January 1991 to 27 December 2005. In the third stage, the cross-sectional regressions estimated using Equation (7) of realised excess portfolio returns on the model predicted returns is performed in each week.

[Insert Table 5 about here]

Table 5 reports the results of the cross-sectional tests with portfolio returns. For the equally weighted size portfolios shown in Panel A, the linear market model has an insignificant intercept $\lambda_0$, a highly significant $\lambda_1$ of 0.89 and an adjusted $R^2$ of 18.65%. Both the quadratic and cubic market models have intercepts that are significant at the 5% level and also both have smaller $\lambda_1$'s than the linear model. The results of the value-weighted size sorts are very similar to those of the equally weighted portfolios.

Panel B of Table 5 shows the results for the momentum sorts. For the equally weighted portfolios, all the models have insignificant intercepts and the linear market model displays a significant and the highest $\lambda_1$ estimate of 0.83 among all the models. Neither the quadratic nor the cubic market model exhibits a greater ability in predicting momentum portfolio returns. The results for the value-weighted portfolios are similar to those of the equally weighted portfolios, though all three models have lower adjusted $R^2$'s. For the equally weighted country portfolios, Panel C of Table 5 indicates that only the linear market model has an insignificant intercept. All the models show a very similar level of performance for the value-weighted country portfolios.

Overall, the linear market model has an insignificant intercept in every case. Neither the quadratic model nor the cubic market model estimates indicate consistent evidence of a greater ability in predicting the one-period-ahead portfolio returns of size, momentum and country sorts. Even so, the magnitudes of the estimated slope coefficients and model adjusted $R^2$s presented in Table 5 are much higher than those estimated in respect of the individual stocks (see Table 4). This suggests that portfolio formations, due to reduced errors in coefficient estimation caused by return outliers of individual stocks and decreases in idiosyncratic risk, improve model performance. Also, according to Kraus and Litzenberg (1973), asymptotic bias is reduced by portfolio formation when measurement errors are less than perfectly correlated.

## *5.2 Time-series results of realised returns on predicted returns*

*Individual stocks*

Table 6 shows the results of the time-series tests with individual stocks. A time-series regression as detailed in Equation (10) of realised excess stock returns on predicted returns is performed for each stock in the third stage. The linear market model has a statistically significant intercept, but has the highest $\lambda_1$ coefficient among all the models. The cubic market model has the lowest $\lambda_1$ and model adjusted $R^2$. These results do not indicate evidence of a greater ability of the non-linear market models in predicting one-period-ahead stock returns.

[Insert Table 6 about here]

Comparing the results presented in Table 6 to those in Table 4 for individual stocks, we see that although relying on the same information, time-series tests generate better model performance than cross-sectional tests. This is because the regression is conducted in time-series to obtain $\lambda_0$ *and* $\lambda_1$ for each stock and then these $\lambda$'s are averaged across stocks and thus the results are less noisy than those obtained from the cross-sectional tests where idiosyncratic risk across stocks is much higher.

*Portfolios*

Table 7 reports the results from the time-series tests with portfolio returns. Panel A of Table 7 shows the results for the size portfolios. The linear market model has a statistically insignificant intercept and a $\lambda_1$ coefficient that is the closest to unity among all the models. Both the quadratic and the cubic market models have statistically significant intercepts and a very similar level of overall performance. For both equally and value-weighted momentum sorts in Panel B of Table 7, all three models show a similar overall performance. Panel C presents the results of the country sorts. For both equally-weighted and value-weighted portfolios, the linear market model has the highest and the most significant $\lambda_1$ estimate among all the models. The cubic market model has the lowest $\lambda_1$ estimate and model adjusted $R^2$. Overall the results of the tests of the size, momentum and country portfolios, indicate that the linear market model performs best among the three models. Comparing the results to those presented in Table 5 for the cross-sectional tests with portfolio returns, the time-series results display much higher $\lambda_1$s and model adjusted $R^2$s than the cross-sectional results.

[Insert Table 7 about here]

## *5.3 Pooled regressions results of realised returns on predicted returns*

*Individual stocks*

Table 8 reports the results of the pooled regression tests with individual stock returns. A pooled regression as detailed in Equation (13) of realised excess returns on predicted returns of all stocks is performed in this third stage test. The linear market model has the highest and significant $\lambda_1$ of 0.79, a significant but the smallest intercept $\lambda_0$ and the highest adjusted $R^2$ of 4.87% of all the models. The quadratic market model has a larger intercept, a smaller

$\lambda_1$ and a lower adjusted $R^2$ than the linear model. The cubic market model has even lower predictive power than the quadratic model. The inclusion of the cubic term seems only to add noise and to reduce the magnitudes of $\lambda_1$ and the adjusted $R^2$. Overall for the tests conducted at the individual stock level, the results presented in Table 4, 6 and 8 provide no evidence that the non-linear market models have a greater ability to predict one-period-ahead returns than the linear CAPM.

[Insert Table 8 about here]

*Portfolios*

Table 9 presents the results of the pooled regressions using portfolio returns. For both the equally and value-weighted size portfolios in Panel A, the linear model and the quadratic market model have very similar performance levels while the cubic market model has the lowest value of $\lambda_1$ among the three models. For the value-weighted momentum portfolios in Panel B, all three models have insignificant intercepts. The linear model shows a slightly higher $\lambda_1$ estimate than the quadratic market model. The cubic market model has the lowest $\lambda_1$ and adjusted $R^2$. For equally weighted momentum portfolios, both the linear and the quadratic models have similar performance, but the cubic market model performs the worst. For country portfolios in Panel C, all three models have insignificant intercepts. The linear market model provides the best performance and the cubic market model has the lowest predictive power.

[Insert Table 9 about here]

## 6. Robustness of the results

### *6.1 Subsample analysis*

This section examines the U.S. sub-sample, which has the largest number of stocks and total market value of all the 19 countries in the sample. This US analysis allows the tests to relax the implicit assumption of market integration and disentangles any exchange rate effects upon returns that the previous tests have ignored. The value-weighted U.S. market portfolio is constructed from the sample stocks in the U.S. markets. The time-series returns of each of the 100 size and momentum portfolios were obtained during the 930 weeks from 8 March 1988 to 27 December 2005. The cross-sectional, time-series and pooled regressions were performed for size and momentum portfolios during the 780 weeks from 22 January 1991 to 27 December 2005.

Table 10 shows the cross-sectional results for the U.S. sample. For both the equally- and value-weighted size portfolios, Panel A shows that although all three models have similar adjusted $R^2$s, the linear market model has an insignificant intercept and also the highest $\lambda_1$. The cubic market model performs worst with the most significant intercept and the lowest $\lambda_1$ among all the models. The results for both the equally and value-weighted momentum portfolios are presented in Panel B and show that the linear market model has the highest $\lambda_1$ among all three models.

[Insert Table 10 about here]

Table 11 shows the time-series test results for the U.S. sample. Overall the results confirm the findings from the whole sample that, for both the equally and value-weighted portfolios sorted by size and momentum, the quadratic and the cubic market models do not appear to perform better than the linear market model. Table 12 shows the results of the pooled regression tests for the U.S. sample. Overall the results again confirm the findings from the whole sample that, for both the equally and value-weighted portfolios sorted by size and momentum, the quadratic and the cubic market models do not outperform the linear market model in predicting one-period-ahead return.

[Insert Table 11 about here]

[Insert Table 12 about here]

### *6.2 Parameter uncertainty*

Overall, the results indicate that the square of the market return deviation contributes incremental explanatory power to the linear CAPM for contemporaneous time-series returns on both the winner and the small size portfolios. However, as shown in Section 5, the higher-moment CAPM market models do not perform well in predicting the one-period-ahead returns. The discrepancy between the explanatory power and predictability of models may reflect higher parameter uncertainty which impinges on forecast accuracy at weekly data frequencies. Parameter uncertainty stems from the fact that the true parameters of a given return model are unknown and must be estimated or inferred from observed information (Williams, 1977; Breanan and Xia, 2001). However, the limited availability and distributional characteristics of observed data introduce random noise and hence hamper precise parameter estimation (DeMiguel and Nogales, 2007). Moreover, the regression coefficients of the return forecasting models may be unstable and subject to changes over time (Paye and Timmermann, 2005). Lewis (2006) also provides evidence of changes in asset pricing relationships over time using weekly stock returns.

Indeed, weekly returns data tend to be more skewed and leptokurtic than monthly data (Brown and Warner, 1985; and Chung et al., 2006) as significant weekly price movements occur more frequently and entail more extreme return observations. By contrast, lower frequency return data tend to smooth out the impacts of extraordinary events happening in a particular period. The point estimates of the slope coefficients of the squared and cubed market terms are more sensitive to the sign and extreme values of realised returns than that of the linear market term. Consequently, the coefficients of the higher-moment terms are estimated with significant uncertainty and perturbations of the estimates induce noisy predictions for one-period-ahead returns. To mitigate the problem, Williams (1977) suggests continuous updating in prior beliefs on parameters to allow for information accumulation over time. The analysis in this paper carries out a rolling-estimation approach to allow for continuous updating of parameter estimates as in Chordia and Shivakumar (2002) for predicting one-period-ahead return.

## 7. Conclusions

Recent literature supports the higher-moment CAPM in pricing stock returns. This paper first poses the question whether the quadratic and cubic market models, respectively consistent with the three- and four-moment CAPM, explain time-series of returns on size and momentum portfolios at weekly data frequencies. The analysis uncovers some interesting findings, in particular, that both the winner and small size portfolios are associated with the market in a non-linear manner and that the squared market return deviation contributes incremental power in explaining the time-variation in returns on these portfolios.

Second, this paper has explored the question as to whether the higher-moment CAPM market models are able to perform better than the linear CAPM in predicting one-period-ahead returns for individual stocks and equally- and value-weighted portfolios of size, momentum and country sorts. The empirical tests adopt cross-sectional, time-series and pooled regressions of realised returns on returns predicted by the models. The answer is surprising but important. The test results using both international and the U.S. data indicate that non-linear market terms do not provide incremental power to the linear CAPM in predicting one-period-ahead returns. The apparently weak roles of non-linear market terms in predicting one-period-ahead returns at weekly data frequency may be due to parameter uncertainty on the quadratic and cubic market factors. Future research could apply different econometric methodologies for comparing results and thus draw more robust conclusions on the return predictability of higher-moment CAPM market models. The framework of

Harvey, et al. (2004) which addresses both parameter uncertainty and higher moments using a posterior predictive approach might represent a new frontier for research in this area.

# Appendix

## *The Four-Moment-Consistent Cubic-Market Model*

This appendix extends the derivation of Kraus and Litzenberger (1976) for the quadratic market model of Equation (3) that is consistent with the three-moment CAPM to the cubic market model. The four-moment CAPM linearly associates the expected return of an asset with the contributions of the asset to the variance, skewness and kurtosis of the market portfolio. The beta, gamma (coskewness scaled by the market skewness) and delta (cokurtosis scaled by the market kurtosis) of risky asset *i* with the market portfolio measure systematic risks,

$$E[R_i] - R_f = \eta_\beta \beta_i + \eta_\gamma \gamma_i + \eta_\delta \delta_i \qquad (14)$$

where $R_f$ is the risk-free rate and $\beta_i$, $\gamma_i$ and $\delta_i$, are defined as in Equation (15). $\eta_\beta$, $\eta_\gamma$ and $\eta_\delta$ are the market prices of beta, gamma and delta, respectively.

$$\begin{aligned} \beta_i &= E\left[\left(R_i - \overline{R}_i\right)\left(R_m - \overline{R}_m\right)\right] / \sigma_m^2 \\ \gamma_i &= E\left[\left(R_i - \overline{R}_i\right)\left(R_m - \overline{R}_m\right)^2\right] / s_m^3 \\ \delta_i &= E\left[\left(R_i - \overline{R}_i\right)\left(R_m - \overline{R}_m\right)^3\right] / k_m^4 \end{aligned} \qquad (15)$$

where $R_{i,}$ and $R_m$ are returns on risky asset *i* and the market portfolio, respectively. $\overline{R}_m$ and $\overline{R}_i$ are respectively mean returns on the market and the asset; $\sigma_m$ , $S_m$ and $K_m$ are respectively the standard deviation, skewness and kurtosis of the market portfolio.

A cubic-market model of the following form is consistent with Equation (14),

$$R_{it} - R_{ft} = C_{0p} + C_{1i}\left(R_{mt} - R_{ft}\right) + C_{2i}\left(R_{mt} - \overline{R}_{mt}\right)^2 + C_{3i}\left(R_{mt} - \overline{R}_{mt}\right)^3 + \varepsilon_t \qquad (16)$$

where $R_{it,}$ $R_{ft}$ and $R_{mt}$ are returns on risky asset $i$, risk-free asset and the market at time $t$ respectively. The notation $\overline{R}_m$ is the mean market return. Express Equation (16) in its deviation form as

$$\left(R_{it} - R_{ft}\right) - E\left(R_{it} - R_{ft}\right) = C_{1i}\left[\left(R_{mt} - R_{ft}\right) - E\left(R_{mt} - R_{ft}\right)\right] \qquad (17)$$

$$+ C_{2i}\left[\left(R_{mt} - \overline{R}_{mt}\right)^2 - E\left(R_{mt} - \overline{R}_{mt}\right)^2\right] + C_{3i}\left[\left(R_{mt} - \overline{R}_{mt}\right)^3 - E\left(R_{mt} - \overline{R}_{mt}\right)^3\right]$$

Multiplying both sides of Equation (17) by $R_{mt} - \overline{R}_{mt}$ , taking expected values and dividing by the variance of the market return, we get the beta of the $i$th risky asset with the market portfolio, $\beta_{i,}$ as

$$\frac{E\left[\left(R_{mt} - \overline{R}_{mt}\right)\left(R_{it} - \overline{R}_{it}\right)\right]}{E\left[\left(R_{mt} - \overline{R}_{mt}\right)^2\right]} = C_{1i} + C_{2i}\frac{E\left[\left(R_{mt} - \overline{R}_{mt}\right)^3\right]}{E\left[\left(R_{mt} - \overline{R}_{mt}\right)^2\right]} + C_{3i}\frac{E\left[\left(R_{mt} - \overline{R}_{mt}\right)^4\right]}{E\left[\left(R_{mt} - \overline{R}_{mt}\right)^2\right]} \qquad (18)$$

Similarly, multiplying both sides of Equation (17) by $\left(R_{mt} - \overline{R}_{mt}\right)^2$, taking expected values and dividing by the third central moment of the market, we obtain the gamma of the $i$th risky asset with the market portfolio, $\gamma_{i,}$ as

$$\frac{E\left[\left(R_{mt} - \overline{R}_{mt}\right)^2\left(R_{it} - \overline{R}_{it}\right)\right]}{E\left[\left(R_{mt} - \overline{R}_{mt}\right)^3\right]} = C_{1i} + C_{2i}\frac{\left[E\left(R_{mt} - \overline{R}_{mt}\right)^4 - \left[E\left(R_{mt} - \overline{R}_{mt}\right)^2\right]^2\right]}{E\left[\left(R_{mt} - \overline{R}_{mt}\right)^3\right]}$$

$$+ C_{3i}\frac{\left[E\left(R_{mt} - \overline{R}_{mt}\right)^5 - E\left(R_{mt} - \overline{R}_{mt}\right)^3 E\left(R_{mt} - \overline{R}_{mt}\right)^2\right]}{E\left[\left(R_{mt} - \overline{R}_{mt}\right)^3\right]} \qquad (19)$$

Finally, multiplying both sides of Equation (17) by $\left(R_{mt} - \overline{R}_{mt}\right)^3$, taking expected values and dividing by the fourth central moment of the market, we obtain the delta of the $i$th risky asset with the market portfolio, $\delta_{i,}$ as

$$\frac{E\left[\left(R_{mt}-\overline{R}_{mt}\right)^3\left(R_{it}-\overline{R}_{it}\right)\right]}{E\left[\left(R_{mt}-\overline{R}_{mt}\right)^4\right]}=C_{1i}+C_{2i}\frac{\left[E\left(R_{mt}-\overline{R}_{mt}\right)^5-E\left(R_{mt}-\overline{R}_{mt}\right)^2E\left(R_{mt}-\overline{R}_{mt}\right)^3\right]}{E\left[\left(R_{mt}-\overline{R}_{mt}\right)^4\right]}$$

$$+C_{3i}\frac{\left[E\left(R_{mt}-\overline{R}_{mt}\right)^6-\left[E\left(R_{mt}-\overline{R}_{mt}\right)^3\right]^2\right]}{E\left[\left(R_{mt}-\overline{R}_{mt}\right)^4\right]} \qquad (20)$$

The left hand side expressions of Equations (18), (19) and (20) are definitions of $\beta_{i,}\ \gamma_i$ and $\delta_i$ as shown in Equation (15) of the four-moment Capital Asset Pricing Model.

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# Table 1

## Descriptive Statistics of the International Sample

Panel A presents for each country the number of stocks, mean and median market values of firms in USD millions at the end of 1987, 1996 and 2005. Panel B and C show, respectively, summary statistics of weekly returns in US dollar of the value-weighted and the equally-weighted global market portfolio (constructed from all sample stocks) and country returns during the 954 weeks from 22 September 1987 to 27 December 2005. 'Euro-$ 1W' denotes the Euro dollar one-week rate. Weekly returns are displayed in percentage. The mean, median, standard deviation (*Stdev*), maximum, minimum, skewness and kurtosis of portfolio returns are calculated for the entire period. The asterisk of *** denotes the significance at 1% level for Jarque-Bera test that compares J.-B. statistics to $\chi^2$ statistics with 2 degrees of freedom for testing whether a distribution is significantly different from normal.

Panel A. Number and Market Value of Firms in Country

| | Number of Firms | | | Mean Market Value ($ M) | | | Median Market Value ($ M) | | |
|---|---|---|---|---|---|---|---|---|---|
| Country | End of 1987 | End of 1996 | End of 2005 | End of 1987 | End of 1996 | End of 2005 | End of 1987 | End of 1996 | End of 2005 |
| Overall | 4,603 | 8,591 | 11,564 | 1,001 | 1,520 | 2,611 | 187 | 214 | 304 |
| Canada | 195 | 455 | 579 | 505 | 713 | 1,807 | 112 | 98 | 214 |
| U.S. | 1,779 | 3,096 | 3,940 | 872 | 1,851 | 3,628 | 125 | 205 | 428 |
| Belgium | 62 | 81 | 141 | 238 | 902 | 2,044 | 63 | 121 | 188 |
| Denmark | 24 | 75 | 84 | 222 | 430 | 1,271 | 153 | 145 | 341 |
| Finland | 3 | 59 | 118 | 151 | 577 | 1,500 | 20 | 91 | 155 |
| France | 87 | 467 | 595 | 887 | 1,043 | 3,064 | 353 | 113 | 190 |
| Germany | 149 | 402 | 634 | 1,002 | 1,392 | 2,035 | 255 | 161 | 155 |
| Italy | 112 | 179 | 323 | 719 | 1,096 | 2,953 | 183 | 173 | 366 |
| Netherlands | 104 | 135 | 182 | 730 | 2,522 | 3,125 | 46 | 222 | 380 |
| Norway | 39 | 109 | 186 | 159 | 370 | 962 | 51 | 111 | 136 |
| Spain | 45 | 107 | 155 | 1,032 | 1,631 | 4,646 | 366 | 409 | 929 |
| Sweden | 8 | 63 | 169 | 331 | 443 | 839 | 82 | 104 | 158 |
| Switzerland | 11 | 41 | 101 | 120 | 208 | 555 | 93 | 117 | 261 |
| U.K. | 878 | 1,400 | 1,542 | 502 | 995 | 1,761 | 88 | 122 | 151 |
| Australia | 100 | 349 | 411 | 457 | 603 | 1,438 | 213 | 124 | 222 |
| Hong Kong | 80 | 281 | 422 | 657 | 1,226 | 1,873 | 190 | 150 | 151 |
| Japan | 803 | 936 | 1,263 | 2,367 | 2,705 | 2,889 | 699 | 788 | 618 |
| Singapore | 97 | 169 | 328 | 178 | 810 | 603 | 65 | 195 | 95 |
| Taiwan | 27 | 187 | 391 | 1,075 | 1,074 | 972 | 426 | 368 | 214 |

| Panel B. Value-Weighted Country Returns, 954 Weeks from 22 September 1987 to 27 December 2005 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Country | *Mean* | *Median* | *Stdev* | Max | Min | Skew | Kurt | J. B. Test |
| The Global Market | 0.17 | 0.28 | 2.00 | 7.61 | -20.12 | -1.33 | 14.71 | *** |
| Canada | 0.21 | 0.37 | 2.17 | 10.33 | -20.66 | -1.37 | 13.49 | *** |
| The U.S. | 0.24 | 0.44 | 2.36 | 12.84 | -24.04 | -1.23 | 16.11 | *** |
| Belgium | 0.22 | 0.31 | 2.37 | 9.95 | -13.37 | -0.44 | 5.67 | *** |
| Denmark | 0.24 | 0.30 | 2.23 | 9.61 | -11.30 | -0.31 | 4.71 | *** |
| Finland | 0.30 | 0.27 | 4.41 | 20.65 | -20.11 | -0.14 | 5.51 | *** |
| France | 0.24 | 0.33 | 2.40 | 10.52 | -12.30 | -0.29 | 5.09 | *** |
| Germany | 0.18 | 0.30 | 2.60 | 10.38 | -12.80 | -0.52 | 5.53 | *** |
| Italy | 0.17 | 0.26 | 3.04 | 12.50 | -11.47 | -0.06 | 4.27 | *** |
| Netherlands | 0.21 | 0.32 | 2.16 | 10.29 | -18.09 | -1.00 | 10.26 | *** |
| Norway | 0.26 | 0.34 | 3.03 | 17.23 | -24.82 | -1.07 | 13.45 | *** |
| Spain | 0.22 | 0.29 | 2.71 | 10.56 | -15.85 | -0.33 | 5.58 | *** |
| Sweden | 0.18 | 0.20 | 3.30 | 26.16 | -16.19 | 0.17 | 9.17 | *** |
| Switzerland | 0.16 | 0.28 | 2.03 | 6.55 | -11.99 | -0.53 | 5.29 | *** |
| The U.K. | 0.18 | 0.25 | 2.07 | 8.86 | -21.10 | -1.24 | 14.98 | *** |
| Australia | 0.21 | 0.27 | 2.58 | 11.90 | -30.53 | -2.01 | 26.25 | *** |
| Hong Kong | 0.29 | 0.46 | 3.86 | 19.54 | -31.78 | -1.07 | 13.20 | *** |
| Japan | 0.09 | 0.05 | 3.13 | 16.58 | -17.79 | 0.10 | 5.67 | *** |
| Singapore | 0.17 | 0.21 | 3.12 | 13.06 | -33.41 | -1.40 | 18.81 | *** |
| Taiwan | 0.25 | 0.49 | 4.90 | 19.94 | -23.85 | -0.26 | 4.67 | *** |
| Euro-$ 1W | 0.09 | 0.10 | 0.04 | 0.21 | 0.02 | 0.03 | 2.37 | *** |

Panel C. Equally-Weighted Countries Returns, 954 Weeks from 22 September 1987 to 27 December 2005

| Country | *Mean* | *Median* | *Stdev* | Max | Min | Skew | Kurt | J. B. Test |
|---|---|---|---|---|---|---|---|---|
| The Global Market | 0.32 | 0.45 | 1.68 | 6.52 | -18.99 | -2.01 | 21.90 | *** |
| Canada | 0.46 | 0.58 | 1.96 | 10.55 | -22.09 | -1.89 | 22.59 | *** |
| The U.S. | 0.42 | 0.68 | 2.32 | 10.13 | -24.41 | -1.59 | 18.13 | *** |
| Belgium | 0.23 | 0.27 | 1.77 | 6.07 | -6.97 | -0.29 | 3.88 | *** |
| Denmark | 0.29 | 0.36 | 1.69 | 7.13 | -10.29 | -0.28 | 5.16 | *** |
| Finland | 0.26 | 0.23 | 2.52 | 13.91 | -13.33 | 0.06 | 6.68 | *** |
| France | 0.32 | 0.33 | 1.89 | 7.90 | -11.27 | -0.42 | 5.48 | *** |
| Germany | 0.23 | 0.23 | 1.89 | 8.65 | -9.99 | -0.42 | 5.53 | *** |
| Italy | 0.18 | 0.25 | 2.53 | 11.55 | -10.64 | -0.04 | 4.74 | *** |
| Netherlands | 0.27 | 0.40 | 1.93 | 8.23 | -15.41 | -0.90 | 8.73 | *** |
| Norway | 0.33 | 0.37 | 2.31 | 9.16 | -16.77 | -0.59 | 7.82 | *** |
| Spain | 0.23 | 0.22 | 2.49 | 16.36 | -13.09 | 0.05 | 7.56 | *** |
| Sweden | 0.29 | 0.32 | 2.78 | 14.97 | -16.33 | -0.28 | 6.68 | *** |
| Switzerland | 0.18 | 0.29 | 1.91 | 6.53 | -11.46 | -0.47 | 5.43 | *** |
| The U.K. | 0.19 | 0.30 | 1.79 | 7.70 | -15.06 | -1.06 | 11.12 | *** |
| Australia | 0.34 | 0.38 | 2.34 | 10.88 | -23.65 | -1.85 | 19.54 | *** |
| Hong Kong | 0.35 | 0.52 | 3.95 | 20.64 | -34.08 | -1.30 | 15.70 | *** |
| Japan | 0.24 | 0.22 | 3.41 | 17.03 | -16.25 | 0.18 | 6.15 | *** |
| Singapore | 0.27 | 0.15 | 3.41 | 17.05 | -22.85 | -0.15 | 9.12 | *** |
| Taiwan | 0.41 | 0.80 | 4.89 | 20.61 | -25.04 | -0.44 | 4.95 | *** |

# Table 2

## Time-Series Explanatory Variables of the Weekly International Sample

Panel A shows mean values of explanatory variables of Equation (1) for the period from 8 March 1988 to 27 December 2005. $R_m$ is the value-weighted return in US dollar on the global market portfolio constructed by using all sample stocks. $R_f$ is the London Financial Times Euro dollar one-week rate that serves as a proxy for the risk free rate. Weekly returns are displayed in percentage. Panel B and C show correlation coefficients and Spearman's rank correlations between explanatory variables, respectively. The *p*-values of correlation coefficients are displayed in parentheses.

Panel A. Mean Values of Variables

| $(R_m - R_f)$ | $(R_m - \overline{R}_m)^2$ | $(R_m - \overline{R}_m)^3$ |
|---|---|---|
| 0.08 | 0.04 | -0.001 |

Panel B. Correlation Coefficients

| | $(R_m - \overline{R}_m)^2$ | $(R_m - \overline{R}_m)^3$ |
|---|---|---|
| $(R_m - R_f)$ | -0.36 | 0.43 |
| *p*-value | (0.000) | (0.000) |
| $(R_m - \overline{R}_m)^2$ | - | -0.90 |
| *p*-value | - | (0.000) |

Panel C. Spearman's Rank Correlation Coefficients

| | $(R_m - \overline{R}_m)^2$ | $(R_m - \overline{R}_m)^3$ |
|---|---|---|
| $(R_m - R_f)$ | -0.023 | 0.999 |
| *p*-value | (0.48) | (0.000) |
| $(R_m - \overline{R}_m)^2$ | 1 | -0.023 |
| *p*-value | | (0.48) |

# Table 3

## Competing Models in Time-Series Regressions of Portfolio Returns

Time-series regressions are performed for evaluating models in explaining returns of the winner and loser deciles, and the smallest and the largest size deciles. The *p*-values of slope coefficients in parentheses are calculated by applying the Newey-West heteroskedasticity-and-autocorrelation-consistent standard errors. The *F*-stat is for testing whether the inclusion of additional explanatory variables to the linear CAPM increases explanatory power. The asterisk of *** denotes the significance at 1% level.

$$R_{pt} - R_{ft} = C_{0p} + C_{1p}\left(R_{mt} - R_{ft}\right) + C_{2p}\left(R_{mt} - \overline{R}_{mt}\right)^2 + C_{3p}\left(R_{mt} - \overline{R}_{mt}\right)^3 + \varepsilon_t \qquad (1)$$

Panel A. Explaining Returns of the Winner and Loser Portfolios, 930 Weeks from 8 March 1988 to 27 December 2005

| Model | | $C_0$ | $C_1$ | $C_2$ | $C_3$ | Adj. $R^2$ | *F*-stat |
|---|---|---|---|---|---|---|---|
| 1 | Winner | 0.003 | 0.902 | | | 0.5508 | - |
| | | (0.000) | (0.000) | | | | |
| | Loser | 0.003 | 1.037 | | | 0.4918 | - |
| | | (0.003) | (0.000) | | | | |
| 2 | Winner | 0.005 | 0.871 | -5.26 | | 0.5737 | 37.23*** |
| | | (0.000) | (0.000) | (0.000) | | | |
| | Loser | 0.003 | 1.030 | -1.280 | | 0.4922 | 1.73 |
| | | (0.000) | (0.000) | (0.756) | | | |
| 3 | Winner | 0.003 | 0.881 | | 12.914 | 0.5509 | 0.52 |
| | | (0.000) | (0.000) | | (0.682) | | |
| | Loser | 0.003 | 0.983 | | 33.622 | 0.4927 | 2.67 |
| | | (0.002) | (0.000) | | (0.215) | | |
| 4 | Winner | 0.005 | 0.900 | -5.506 | -18.863 | 0.5739 | 19.07*** |
| | | (0.000) | (0.000) | (0.000) | (0.447) | | |
| | Loser | 0.003 | 0.986 | -0.914 | 28.344 | 0.4926 | 1.74 |
| | | (0.000) | (0.000) | (0.509) | (0.352) | | |

Panel B. Explaining Returns of the Small and Big portfolios, 930 Weeks from 8 March 1988 to 27 December 2005

| Model | | $C_0$ | $C_1$ | $C_2$ | $C_3$ | Adj. $R^2$ | *F*-Test |
|---|---|---|---|---|---|---|---|
| 1 | Small | 0.005 | 0.425 | | | 0.2990 | - |
| | | (0.000) | (0.000) | | | | |
| | Big | 0.000 | 0.985 | | | 0.9525 | - |
| | | (0.136) | (0.000) | | | | |
| 2 | Small | 0.006 | 0.406 | -3.243 | | 0.3211 | 60.79*** |
| | | (0.000) | (0.000) | (0.000) | | | |
| | Big | 0.000 | 0.989 | 0.573 | | 0.9529 | 0.38 |
| | | (0.888) | (0.000) | (0.124) | | | |
| 3 | Small | 0.005 | 0.423 | | 1.441 | 0.2993 | 0.03 |
| | | (0.000) | $(4\times10^{-33})$ | | (0.928) | | |
| | Big | 0.000 | 0.993 | | -4.795 | 0.9526 | 0.06 |
| | | (0.165) | (0.000) | | (0.516) | | |
| 4 | Small | 0.006 | 0.435 | -3.487 | -18.795 | 0.3219 | 32.3*** |
| | | (0.000) | (0.000) | (0.000) | (0.250) | | |
| | Big | 0.000 | 0.991 | 0.553 | -1.587 | 0.9528 | 0.19 |
| | | (0.881) | (0.000) | (0.136) | (0.818) | | |

# Table 4

## Cross-Sectional Regressions for Realised on Predicted Returns of Individual Stocks

The Table reports the results of cross-sectional regressions (Equation 7) of realised excess returns on model predicted returns for 12,262 individual stocks that have at least 162 weeks return history. The reported intercept and slope coefficient, $\lambda_j$, are averages across all cross-sectional periods according to Equation (8). The *p*-value displayed in parentheses for testing the significance of each coefficient is the *p*-value corresponding to the *t*-statistic calculated as in Equation (9).

$$\tilde{r}_{i,t+1} = \lambda_0 + \lambda_1 \hat{r}_{i,t+1} + \tilde{\varepsilon}_i \tag{7}$$

$$\lambda_j = \frac{1}{T}\sum_{t=1}^{T}\lambda_{jt} \quad \text{where } j = 0 \text{ and } 1\text{, and } T \text{ is the number of cross-sectional periods,} \tag{8}$$

$$t_{\lambda j} = \frac{\lambda_j}{std_{\lambda j} / \sqrt{T}} \quad \text{where } std_{\eta j} \text{ is the standard deviation of } \lambda_j. \tag{9}$$

To compute model predicted stock returns, the intercept and slope coefficients of Equations (1), (3) and (5) are estimated each week on a rolling basis during the 780 weeks from 15 January 1991 to 20 December 2005 for each stock by using previous 150 weeks of returns in U.S. dollar on stocks and the value-weighted global market portfolio computed using all sample stocks. Once loadings are estimated, they are used to predict one-period-ahead stock returns by incorporating realised market return into Equations (2), (4) and (6) for the cubic-, quadratic- and linear-market models, respectively.

| | 780 Weeks from 22 January 1991 to 27 December 2005 | | |
|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0013 | 0.3898 | 0.0199 |
| | (0.000) | (0.000) | |
| Quadratic Market Model | 0.0017 | 0.3850 | 0.0189 |
| | (0.000) | (0.000) | |
| Cubic Market Model | 0.0016 | 0.3495 | 0.0174 |
| | (0.000) | (0.000) | |

# Table 5

## Cross-Sectional Regressions of Realised on Predicted Portfolio Returns

Panel A, B, and C show, respectively, the results of cross-sectional regressions of realised excess returns on model predicted returns for 100 size, 100 momentum and 19 country portfolios. The *p*-values for testing coefficient significance are displayed in parentheses.

Equally and value-weighted portfolio returns are obtained during the 930 weeks from 8 March 1988 to 27 December 2005. To compute model predicted portfolio returns, the intercept and slope coefficients of Equations (1), (3) and (5) are estimated each week on a rolling basis during the 780 weeks from 15 January 1991 to 20 December 2005 for each portfolio by using previous 150 weeks of returns in U.S. dollar on portfolios and the value-weighted global market portfolio computed using all sample stocks. Once loadings are estimated, they are used to predict one-period-ahead portfolio returns by incorporating realised market return into Equations (2), (4) and (6) for the cubic-, quadratic- and linear-market models, respectively.

Panel A. 100 Size Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005

| | Equally Weighted | | | Value-Weighted | | |
|---|---|---|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0006 | 0.8928 | 0.1865 | 0.0006 | 0.8897 | 0.1867 |
| | (0.358) | (0.000) | | (0.355) | (0.000) | |
| Quadratic Market Model | 0.0015 | 0.8145 | 0.1906 | 0.0015 | 0.8123 | 0.1907 |
| | (0.014) | (0.000) | | (0.015) | (0.000) | |
| Cubic Market Model | 0.0012 | 0.8196 | 0.1879 | 0.0012 | 0.8183 | 0.1882 |
| | (0.036) | (0.000) | | (0.039) | (0.000) | |

Panel B. 100 Momentum Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005

| | Equally Weighted | | | Value-Weighted | | |
|---|---|---|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | -0.0001 | 0.8316 | 0.1233 | -0.0004 | 0.5725 | 0.0781 |
| | (0.920) | (0.000) | | (0.696) | (0.000) | |
| Quadratic Market Model | 0.0000 | 0.7588 | 0.1254 | 0.0002 | 0.5226 | 0.0823 |
| | (0.976) | (0.000) | | (0.833) | (0.000) | |
| Cubic Market Model | 0.0004 | 0.6973 | 0.1279 | 0.0006 | 0.4444 | 0.0794 |
| | (0.575) | (0.000) | | (0.459) | (0.000) | |

Panel C. 19 Country Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005

| | Equally Weighted | | | Value-Weighted | | |
|---|---|---|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0005 | 0.5223 | 0.0823 | 0.0003 | 0.5622 | 0.0698 |
| | (0.384) | (0.000) | | (0.682) | (0.000) | |
| Quadratic Market Model | 0.0013 | 0.5338 | 0.0873 | 0.0006 | 0.5721 | 0.0680 |
| | (0.052) | (0.000) | | (0.373) | (0.000) | |
| Cubic Market Model | 0.0012 | 0.5273 | 0.0917 | 0.0006 | 0.5421 | 0.0740 |
| | (0.085) | (0.000) | | (0.435) | (0.000) | |

# Table 6

## Time-Series Regressions of Realised on Predicted Individual Stock Returns

The Table reports the results of time-series regressions (Equation 10) of realised excess returns on model predicted returns for 12,262 individual stocks that have at least 162-week return history. The reported intercept and slope coefficient, $\lambda_j$, are averages across portfolios as in Equation (11). The *p*-value displayed in parentheses for testing the significance of each coefficient is the *p*-value corresponding to the *t*-statistic calculated as in Equation (12).

$$\tilde{r}_{i,t+1} = \lambda_0 + \lambda_1 \hat{r}_{i,t+1} + \varepsilon_{t+1} \tag{10}$$

$$\lambda_j = \frac{1}{N}\sum_{i=1}^{N} \lambda_{ji} \quad \text{where } j = 0 \text{ and } 1, \text{ and } N \text{ is the number of stocks.} \tag{11}$$

$$t_{\lambda j} = \frac{\lambda_j}{std_{\lambda j} / \sqrt{N}} \quad \text{where } std_{\lambda j} \text{ is the standard deviation of } \lambda_j. \tag{12}$$

To compute model predicted stock returns, the intercept and slope coefficients of Equations (1), (3) and (5) are estimated each week on a rolling basis during the 780 weeks from 15 January 1991 to 20 December 2005 for each stock by using previous 150 weeks of returns in U.S. dollar on stocks and the value-weighted global market portfolio computed using all sample stocks. Once loadings are estimated, they are used to predict one-period-ahead stock returns by incorporating realised market return into Equations (2), (4) and (6) for the cubic-, quadratic- and linear-market models, respectively.

| | 780 Weeks from 22 January 1991 to 27 December 2005 | | |
|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0011 | 0.6189 | 0.0570 |
| | (0.000) | (0.000) | |
| Quadratic Market Model | 0.0012 | 0.5598 | 0.0525 |
| | (0.000) | (0.000) | |
| Cubic Market Model | 0.0015 | 0.4570 | 0.0470 |
| | (0.000) | (0.000) | |

# Table 7

## Time-series Regressions of Realised on Predicted Portfolio Returns

Panel A, B, and C show, respectively, the results from time-series regressions of realised excess returns on model predicted returns for 100 size, 100 momentum and 19 country portfolios. The *p*-values for testing coefficient significance are displayed in parentheses.

Equally and value-weighted portfolio returns are obtained during the 930 weeks from 8 March 1988 to 27 December 2005. To compute model predicted portfolio returns, the intercept and slope coefficients of Equations (1), (3) and (5) are estimated each week on a rolling basis during the 780 weeks from 15 January 1991 to 20 December 2005 for each portfolio by using previous 150 weeks of returns in U.S. dollar on portfolios and the value-weighted global market portfolio computed using all sample stocks. Once loadings are estimated, they are used to predict one-period-ahead portfolio returns by incorporating realised market return into Equations (2), (4) and (6) for the cubic-, quadratic- and linear-market models, respectively.

Panel A. 100 Size Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005

| | Equally Weighted | | | Value-Weighted | | |
|---|---|---|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0000 | 0.9853 | 0.5666 | 0.0000 | 0.9856 | 0.5664 |
| | (0.734) | (0.000) | | (0.618) | (0.000) | |
| Quadratic Market Model | 0.0000 | 0.9683 | 0.5760 | 0.0000 | 0.9683 | 0.5758 |
| | (0.000) | (0.000) | | (0.000) | (0.000) | |
| Cubic Market Model | 0.0001 | 0.9442 | 0.5749 | 0.0001 | 0.9442 | 0.5747 |
| | (0.000) | (0.000) | | (0.000) | (0.000) | |

Panel B. 100 Momentum Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005

| | Equally Weighted | | | Value-Weighted | | |
|---|---|---|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0001 | 0.9814 | 0.5136 | -0.0001 | 0.9701 | 0.4750 |
| | (0.000) | (0.000) | | (0.012) | (0.000) | |
| Quadratic Market Model | 0.0001 | 0.9733 | 0.5184 | -0.0001 | 0.9505 | 0.4708 |
| | (0.000) | (0.000) | | (0.000) | (0.000) | |
| Cubic Market Model | 0.0002 | 0.9402 | 0.5127 | 0.0000 | 0.9010 | 0.4531 |
| | (0.000) | (0.000) | | (0.139) | (0.000) | |

Panel C. 19 Country Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005

| | Equally Weighted | | | Value-Weighted | | |
|---|---|---|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0000 | 0.9443 | 0.2633 | 0.0000 | 0.9720 | 0.3855 |
| | (0.882) | (0.000) | | (0.921) | (0.000) | |
| Quadratic Market Model | 0.0000 | 0.9186 | 0.2629 | 0.0000 | 0.9488 | 0.3798 |
| | (0.782) | (0.000) | | (0.782) | (0.000) | |
| Cubic Market Model | 0.0001 | 0.8411 | 0.2444 | 0.0001 | 0.9045 | 0.3669 |
| | (0.335) | (0.000) | | (0.415) | (0.000) | |

# Table 8

## Pooled Regressions of Realised on Predicted Individual Stock Returns

The Table reports the results of a pooled regression (Equation 13) of realised excess returns on model predicted returns for 12,262 individual stocks that have at least 162-week return history. The *p*-values for testing coefficient significance are displayed in parentheses.

$$\tilde{r}_{i,t+1} = \lambda_0 + \lambda_1 \hat{r}_{i,t+1} + \tilde{\varepsilon}_{i,t+1} \tag{13}$$

To compute model predicted stock returns, the intercept and slope coefficients of Equations (1), (3) and (5) are estimated each week on a rolling basis during the 780 weeks from 15 January 1991 to 20 December 2005 for each stock by using previous 150 weeks of returns in U.S. dollar on stocks and the value-weighted global market portfolio computed using all sample stocks. Once loadings are estimated, they are used to predict one-period-ahead stock returns by incorporating realised market return into Equations (2), (4) and (6) for the cubic-, quadratic- and linear-market models, respectively.

| | 780 Weeks from 22 January 1991 to 27 December 2005 | | |
|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0005 | 0.7909 | 0.0487 |
| | (0.000) | (0.000) | |
| Quadratic Market Model | 0.0007 | 0.6851 | 0.0432 |
| | (0.000) | (0.000) | |
| Cubic Market Model | 0.0012 | 0.4990 | 0.0327 |
| | (0.000) | (0.000) | |

## Table 9

### Pooled Regressions of Realised on Predicted Portfolio Returns

Panel A, B, and C show, respectively, the results from a pooled regression (Equation 13) of realised excess returns on model predicted returns for 100 size, 100 momentum and 19 country portfolios. The *p*-values for testing coefficient significance are displayed in parentheses.

$$\tilde{r}_{p,t+1} = \lambda_0 + \lambda_1 \hat{r}_{p,t+1} + \tilde{\varepsilon}_{p,t+1} \tag{13}$$

Equally and value-weighted portfolio returns are obtained during the 930 weeks from 8 March 1988 to 27 December 2005. To compute model predicted portfolio returns, the intercept and slope coefficients of Equations (1), (3) and (5) are estimated each week on a rolling basis during the 780 weeks from 15 January 1991 to 20 December 2005 for each portfolio by using previous 150 weeks of returns in U.S. dollar on portfolios and the value-weighted global market portfolio computed using all sample stocks. Once loadings are estimated, they are used to predict one-period-ahead portfolio returns by incorporating realised market return into Equations (2), (4) and (6) for the cubic-, quadratic- and linear-market models, respectively.

Panel A. 100 Size Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005

| | Equally Weighted | | | Value-Weighted | | |
|---|---|---|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0001 | 0.9862 | 0.5849 | 0.0000 | 0.9862 | 0.5851 |
| | (0.218) | (0.000) | | (0.238) | (0.000) | |
| Quadratic Market Model | 0.0000 | 0.9812 | 0.5920 | 0.0000 | 0.9812 | 0.5921 |
| | (0.802) | (0.000) | | (0.756) | (0.000) | |
| Cubic Market Model | 0.0000 | 0.9576 | 0.5897 | 0.0000 | 0.9574 | 0.5900 |
| | (0.756) | (0.000) | | (0.802) | (0.000) | |

Panel B. 100 Momentum Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005

| | Equally Weighted | | | Value-Weighted | | |
|---|---|---|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0001 | 0.9835 | 0.4836 | -0.0001 | 0.9723 | 0.4535 |
| | (0.028) | (0.000) | | (0.322) | (0.000) | |
| Quadratic Market Model | 0.0001 | 0.9725 | 0.4862 | -0.0001 | 0.9516 | 0.4474 |
| | (0.089) | (0.000) | | (0.109) | (0.000) | |
| Cubic Market Model | 0.0002 | 0.9253 | 0.4732 | 0.0000 | 0.8891 | 0.4242 |
| | (0.000) | (0.000) | | (0.667) | (0.000) | |

Panel C. 19 Country Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005

| | Equally Weighted | | | Value-Weighted | | |
|---|---|---|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0000 | 0.9358 | 0.2416 | 0.0000 | 0.9666 | 0.3487 |
| | (0.992) | (0.000) | | (0.996) | (0.000) | |
| Quadratic Market Model | 0.0000 | 0.8968 | 0.2367 | 0.0000 | 0.9410 | 0.3407 |
| | (0.976) | (0.000) | | (0.841) | (0.000) | |
| Cubic Market Model | 0.0001 | 0.8324 | 0.2242 | 0.0001 | 0.8931 | 0.3276 |
| | (0.555) | (0.000) | | (0.624) | (0.000) | |

## Table 10

### Cross-Sectional Regressions of Realised on Predicted Portfolio Returns of the U.S. Sample

Panel A and B show, respectively, the results from cross-sectional regressions of realised excess returns on model predicted returns for U.S. size and momentum portfolios as:

$$\tilde{r}_{p,t+1} = \lambda_0 + \lambda_1 \hat{r}_{p,t+1} + \tilde{\varepsilon}_p$$

Equally and value-weighted portfolio returns are obtained during the 930 weeks from 8 March 1988 to 27 December 2005. To compute model predicted portfolio returns, the intercept and slope coefficients of Equations (1), (3) and (5) are estimated each week on a rolling basis during the 780 weeks from 15 January 1991 to 20 December 2005 for each portfolio by using previous 150 weeks of returns in U.S. dollar on portfolios and the value-weighted U.S. market portfolio computed using all sample stocks in the U.S. markets. Once loadings are estimated, they are used to predict one-period-ahead portfolio returns by incorporating realised market return into Equations (2), (4) and (6) for the cubic-, quadratic- and linear-market models, respectively. The *p*-values for testing coefficient significance are displayed in parentheses.

Panel A. 100 Size Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005

| | Equally Weighted | | | Value-Weighted | | |
|---|---|---|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0007 | 0.8453 | 0.1030 | 0.0007 | 0.8413 | 0.1022 |
| | (0.447) | (0.000) | | (0.447) | (0.000) | |
| Quadratic Market Model | 0.0015 | 0.7572 | 0.1060 | 0.0015 | 0.7518 | 0.1051 |
| | (0.082) | (0.000) | | (0.085) | (0.000) | |
| Cubic Market Model | 0.0019 | 0.7127 | 0.1054 | 0.0019 | 0.7096 | 0.1045 |
| | (0.022) | (0.000) | | (0.024) | 0.000) | |

Panel B. 100 Momentum Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005

| | Equally Weighted | | | Value-Weighted | | |
|---|---|---|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0004 | 0.6882 | 0.0809 | 0.0008 | 0.5599 | 0.0635 |
| | (0.617) | (0.000) | | (0.459) | (0.000) | |
| Quadratic Market Model | 0.0004 | 0.6380 | 0.0780 | 0.0005 | 0.5330 | 0.0617 |
| | (0.645) | (0.000) | | (0.631) | (0.000) | |
| Cubic Market Model | 0.0005 | 0.5762 | 0.0765 | 0.0004 | 0.4683 | 0.0630 |
| | (0.478) | (0.000) | | (0.711) | (0.000) | |

## Table 11

### Time-series Regressions of Realised on Predicted Portfolio Returns of the U.S. Sample

Panel A and B show, respectively, the results from time-series regressions of realised excess returns on model predicted returns for U.S. size and momentum portfolios as:

$$\tilde{r}_{p,t+1} = \lambda_0 + \lambda_1 \hat{r}_{p,t+1} + \varepsilon_{t+1}.$$

Equally and value-weighted portfolio returns are obtained during the 930 weeks from 8 March 1988 to 27 December 2005. To compute model predicted portfolio returns, the intercept and slope coefficients of Equations (1), (3) and (5) are estimated each week on a rolling basis during the 780 weeks from 15 January 1991 to 20 December 2005 for each portfolio by using previous 150 weeks of returns in U.S. dollar on portfolios and the value-weighted U.S. market portfolio computed using all sample stocks in the U.S. markets. Once loadings are estimated, they are used to predict one-period-ahead portfolio returns by incorporating realised market return into Equations (2), (4) and (6) for the cubic-, quadratic- and linear-market models, respectively. The *p*-values for testing coefficient significance are displayed in parentheses.

Panel A. 100 Size Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005

| | Equally Weighted | | | Value-Weighted | | |
|---|---|---|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0000 | 0.9889 | 0.4922 | 0.0000 | 0.9893 | 0.4922 |
| | (0.283) | (0.000) | | (0.283) | (0.000) | |
| Quadratic Market Model | 0.0005 | 0.9405 | 0.4950 | 0.0005 | 0.9404 | 0.4949 |
| | (0.000) | (0.000) | | (0.000) | (0.000) | |
| Cubic Market Model | 0.0010 | 0.8234 | 0.4510 | 0.0010 | 0.8232 | 0.4508 |
| | (0.000) | (0.000) | | (0.000) | (0.000) | |

Panel B. 100 Momentum Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005

| | Equally Weighted | | | Value-Weighted | | |
|---|---|---|---|---|---|---|
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0002 | 1.0017 | 0.4805 | 0.0001 | 0.9925 | 0.4500 |
| | (0.000) | (0.000) | | (0.156) | (0.000) | |
| Quadratic Market Model | 0.0006 | 0.9635 | 0.4730 | 0.0001 | 0.9557 | 0.4377 |
| | (0.000) | (0.000) | | (0.000) | (0.000) | |
| Cubic Market Model | 0.0009 | 0.8768 | 0.4312 | 0.0003 | 0.8647 | 0.4031 |
| | (0.000) | (0.000) | | (0.000) | (0.000) | |

# Table 12

## Pooled Regressions of Realised on Predicted Portfolio Returns of the U.S. Sample

Panel A and B show, respectively, the results from a pooled regression of realised excess returns on model predicted returns for U.S. size and momentum portfolios as:

$$\tilde{r}_{p,t+1} = \lambda_0 + \lambda_1 \hat{r}_{p,t+1} + \tilde{\varepsilon}_{p,t+1} \,.$$

Equally and value-weighted portfolio returns are obtained during the 930 weeks from 8 March 1988 to 27 December 2005. To compute model predicted portfolio returns, the intercept and slope coefficients of Equations (1), (3) and (5) are estimated each week on a rolling basis during the 780 weeks from 15 January 1991 to 20 December 2005 for each portfolio by using previous 150 weeks of returns in U.S. dollar on portfolios and the value-weighted U.S. market portfolio computed using all sample stocks in the U.S. markets. Once loadings are estimated, they are used to predict one-period-ahead portfolio returns by incorporating realised market return into Equations (2), (4) and (6) for the cubic-, quadratic- and linear-market models, respectively. The *p*-values for testing coefficient significance are displayed in parentheses.

| Panel A. 100 Size Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005 | | | | | | |
|---|---|---|---|---|---|---|
| | Equally Weighted | | | Value-Weighted | | |
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0000 | 0.9974 | 0.4708 | 0.0000 | 0.9976 | 0.4711 |
| | (0.522) | (0.000) | | (0.528) | (0.000) | |
| Quadratic Market Model | 0.0003 | 0.9603 | 0.4715 | 0.0003 | 0.9603 | 0.4717 |
| | (0.000) | (0.000) | | (0.000) | (0.000) | |
| Cubic Market Model | 0.0007 | 0.8444 | 0.4159 | 0.0007 | 0.8444 | 0.4162 |
| | (0.000) | (0.000) | | (0.000) | (0.000) | |

| Panel B. 100 Momentum Portfolios, 780 Weeks from 22 January 1991 to 27 December 2005 | | | | | | |
|---|---|---|---|---|---|---|
| | Equally Weighted | | | Value-Weighted | | |
| | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ | $\lambda_0$ | $\lambda_1$ | Adj. $R^2$ |
| Linear CAPM | 0.0003 | 0.9907 | 0.4348 | 0.0001 | 0.9827 | 0.4255 |
| | (0.000) | (0.000) | | (0.312) | (0.000) | |
| Quadratic Market Model | 0.0006 | 0.9490 | 0.4288 | 0.0002 | 0.9477 | 0.4116 |
| | (0.000) | (0.000) | | (0.087) | (0.000) | |
| Cubic Market Model | 0.0010 | 0.8343 | 0.3738 | 0.0003 | 0.8562 | 0.3770 |
| | (0.000) | (0.000) | | (0.001) | (0.000) | |